Manuscript Number: ASR-D-17-00076

Title: An analytical model of prominence dynamics

Article Type: SI: Waves in Solar Atmosphere

Keywords: Solar Physics; Sun; Solar Prominence; MHD waves

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An analytical model of prominence dynamics

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Abstract

Solar Prominence is an intriguing, but poorly understood magnetic structure of the solar corona. Convective motions in the photo-sphere and sub photo-sphere may be responsible for generating the magnetic fields that support long-lived quiescent solar prominence. The dynamics of solar prominence has been the subject of a large number of studies. We developed an analytic model to analyze the nature of the dynamics of these quiescent solar prominence.

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1. Introduction

It is a well known fact that Solar Prominences are cool, dense plasma clouds, composed of small-scale ever-changing threads of fibrils, embedded in the corona [2]. The prominence plasma is in nearly equilibrium state which is supported by the magnetic field, against gravity [13, 26].

Quiescent Prominences are large and appear as thin vertical sheets endowed with fine filamentary structure. These prominences display very minor changes over a period of time (days)[5]. In spite of the word "Quiescent" attached with it, these prominences display a prominent mass motion when observations are made in high resolution Hα movies. In these filaments, the solar material is

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concentrated as rope-like structures having diameters less than 300 km.

Two main types of topology have been suggested for supporting prominences that are related to magnetic fields. First one was by Kipppenhahn and Schluter in 1957 [13] and second one was by Kuperus and Tandberg-Hanssen in 1967 [14], developed further by Kuperus and Raadu in 1973 [26]. In the Kuperus-Schlitter (K-S) model, the prominence material sits on top of the field lines supported by the normal polarity field. The K-R model suggests that the prominence is embedded in an inverse polarity field. Simply stated, a prominence is considered as a sheet of plasma, standing erect in the corona, above a magnetic neutral line.

Prominence are highly dynamical structures exhibiting flows in Hα, UV and EUV lines. The study of these flows improve our understanding of prominence formation and stability, the mass supply, and the magnetic field structure of the prominence, hence making those topic of great interest. In the Hα lines, and in quiescent limb prominences, a complex dynamics with vertical down flows, up flows and horizontal flows is observed [2]. The velocity of these flows lie between a range of 2 and 35 km/s, while in EUV lines, these flows seem to be of slightly higher velocities. The point to be considered here is that these observations from these lines correspond to various temperatures which would imply that the flow speeds correspond to different parts of the prominence. The velocities have been reported to even reaching 200 km/s in active regions. In the case of filaments observed on the disk in the Hα line, horizontal flows in the filament spine are often observed, while in barbs flows are vertical. The observed range for the filament flow velocities is 5 to 200 km/s. The presence of counter streaming (oppositely directed) flows is also a feature observed from these observations [34, 16]. These flows seem to be field aligned due to the filament plasma. In this paper we tried an analytical approach to analyze prominent filament's dynamics by taking a simplified model (for analytical calculation).

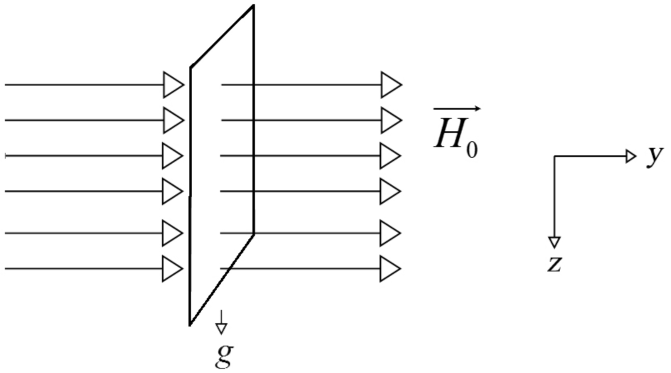

Figure 1: Schematic representation of prominence plate along with conditions

2. MHD equations

Solar prominences are approximated by filaments made up of thin sheets of plasma. (Low, 1981,1982) .Suppose there is a one dimensional infinite vertical rigid sheet of perfectly conducting massive material sitting in a perfectly conducting in-compressible static fluid. Let the sheet be threaded by a uniform magnetic field perpendicular to the sheet; gravity is uniform and is acting vertical to the plate. The plate falls under the action of the gravity. Gravity is otherwise neglected for the medium (assuming the medium's density is much less compared to plate's density) but considered to be acting on the plate. As it descends it will introduce perturbation in the medium. Let $\vec{H}(y,t)\hat{z}$ and $\vec{v} = v(y,t)\hat{z}$ be the perturbations in magnetic field and velocity of the medium. The magnetic field acting on the prominence is described as $\vec{B} = [0, H_0, \vec{H}(y,t)]$, where $\vec{H}(y,t)\hat{z}$ is perturbation in magnetic field, and H_0 is constant magnetic field in y direction, perpendicular to the plate.

The z-component of the MHD momentum equation for the medium outside the prominence sheet can be written in the form,

$$\rho \frac{\partial \vec{v}}{\partial t} = \frac{H_0}{4\pi} \frac{\partial H}{\partial y}. \tag{1}$$

and the z-component of MHD induction equation becomes,

$$\frac{\partial \vec{H}}{\partial t} = H_0 \frac{\partial v}{\partial y}. \tag{2}$$

An important point to consider is we have not used any small amplitude approximation to linearize the MHD equations to obtain the above equations.

2.1. Alfven wave equations

From Eqs. 1 & 2 the following equations are obtained,

$$\frac{\partial^2 v}{\partial t^2} = \frac{H_0^2}{4\pi\rho_0} \frac{\partial^2 v}{\partial y^2} \tag{3}$$

$$\frac{\partial^2 H}{\partial t^2} = \frac{H_0^2}{4\pi\rho_0} \frac{\partial^2 H}{\partial y^2}. \tag{4}$$

The above two equations (3 & 4), are Alfven Wave equations, with Alfven wave velocity $V_A = \frac{H_0}{\sqrt{4\pi\rho_0}}$.

2.2. Solution to Wave Equations

The following initial conditions are considered, Initial Condition : at $t = 0, H = 0$ and $v = 0$. The boundary Condition : at $y = 0, v = u(t)\Big|_{y=0}$, where $u(t)$ is the prominence sheet's velocity.
The general solution for the above wave equation 3 is

$$H = F(y - V_A t) + G(y + V_A t), \tag{5}$$

The general solution of the one-dimensional wave equation is sum of a right traveling function F and a left traveling function G. "Traveling" means that the shape of these individual arbitrary functions with respect to y stays constant, however the functions are translated left and right with time at the speed V_A(Jean le Rond D'Alembert [4]). The solution on substitution in eq. 1 & 2, gives

$$v = \frac{1}{\sqrt{4\pi\rho_0}}[-F(y - V_A t) + G(y + V_A t)]. \tag{6}$$

2.3. Momentum equation for prominence filament

It is imperative to understand the nature of the velocity in order to land credence to the theory of the dynamics of the filaments. MHD momentum equation for the prominence sheet reads

$$m_p \frac{\partial u}{\partial t}\bigg|_{y=0} = -m_p g + \int_{y=0-}^{y=0+} \frac{H_0}{4\pi}\frac{\partial H}{\partial y}\bigg|_{y=0}. \tag{7}$$

where, m_p is the integrated mass density across the thickness of the thin sheet (mass density $\rho\times$ thickness t)

The second boundary condition i.e normal component of the magnetic field being continuous across the plate, renders eq. 7 as,

$$m_p \frac{\partial u}{\partial t}\bigg|_{y=0} = m_p g - \frac{H_0 H}{2\pi}\bigg|_{y=0}. \tag{8}$$

Incorporating the initial conditions in 5 & 6, for t=0, yields $F(y) = G(y)$. The initial condition $t = 0, v = 0$, yields $F(y) = -G(y)$ Thus,

$$\begin{aligned} G(y) &= 0 \\ F(y) &= 0 \end{aligned} \qquad t = 0, y > 0 \tag{9}$$

Now consider, $(y-V_At) = \xi$ and $(y+V_At) = \eta$, similarly, $\xi > 0$ implies $F(\xi) = 0$, $\eta > 0$ implies $G(\eta) = 0$. Also note, for the 2 cases, $\xi > 0$ implies $y > V_At$ implies $F(\xi) = 0$ and $G(\xi) = 0$. This is a trivial solution. And for $\xi < 0$ implies $y < V_At$ implies $F(\xi) \neq .0$, gives us equation 8 in the form,

$$m_p \frac{\mathrm{d}}{\mathrm{dt}}\left[\frac{1}{\sqrt{4\pi\rho_0}}(-F(y-V_At) + G(y+V_At))\right]\bigg|_{y=0} = \tag{10}$$

$$m_p g - \frac{H_0}{2\pi}\Big[F(y-V_At) + G(y+V_At)\Big]\bigg|_{y=0}. \tag{11}$$

Applying boundary condition on eq. 5, yields

$$H = F(-V_At), \tag{12}$$

$$u = -\frac{1}{\sqrt{4\pi\rho}}(F(-V_At), \tag{13}$$

Substituting eq. 12 and eq. 13 in eq. 8, one can obtain,

$$-\frac{m_p}{\sqrt{4\pi\rho}}\frac{\partial F(-V_A t)}{\partial t}\bigg|_{y=0} = -m_p g + \frac{H_0 F(-V_A t)}{2\pi}\bigg|_{y=0}. \tag{14}$$

This is a differential equation of first order, which is solved by using *Integrating Factor*, ([28]) as follows

$$I = exp\int(2\rho_0/m_p)dx. \tag{15}$$

applying this integrating factor, the solution for the equation (eq. 15) is obtained as,

$$u = \frac{m_p g}{2\rho_0 V_A}\left[1 - exp(-\rho_0 V_A t/m_p)\right] \tag{16}$$

Where,

u is the prominence velocity

m_p is the prominence mass density

g is the acceleration due to gravity of the sun

ρ_0 is the density of the medium around the plate / filaments

V_A is the Alfven wave velocity

and t is the time of the perturbation.

3. Results

The parameters are substituted with the following approximated values that from the solar atmosphere viz. H_0 (magnetic Field l) as $20 \times 10^{-4}T$, the acceleration due to gravity on the solar surface, g as $274.2ms^{-2}$, prominence filament density as $5 \times 10^{-11}kgm^{-3}$ which is $300km$ in width making m_p which is prominence mass across the width of the thread (prominence mass density x width of the prominence thread) as $1.5 \times 10^{-5}kg/m^2$, Coronal density $5 \times 10^{-14}kgm^{-3}$, resulting in an Alfven velocity of $7.981 \times 10^6 ms^{-1}$ and the prominence velocity attaining the maximum value of $5.153 \times 10^3 ms^{-1}$.

The result shows an exponential increase in the speed of the prominence thread from the equilibrium state over very short time (few minutes) and after that there is a downward fall (towards solar surface) of the prominence thread with an uniform velocity (almost 5 km/sec).

The prominence velocity vs time is shown in fig. (2). Velocities of bright threads have been observed by the time slice technique and the values were estimated to a few kilometers per second to a maximum of $6kms^{-1}$ The journal A&A published in 2010, reported a finding by *B. Schmieder et al.*, of the velocity of the prominence filaments as,

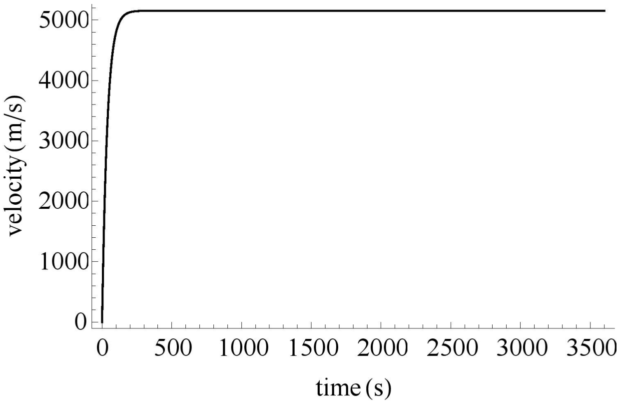

Figure 2: Prominence velocity as a function of time eq(16). The rise in velocity up to 200 seconds approximately and thereafter becomes constant (5.1 km/sec).

> *"Velocities V(x,z) of the bright threads are computed by time slice technique and these values are of the order of a few* kms^{-1} *to 6* kms^{-1} *reaching 11* kms^{-1} *for individual fine threads".*

Our calculation, as mentioned above, of the prominence filament velocity was $5.1kms^{-1}$, which is in agreement with the observations [29].

Our results also show Alfven waves (not necessarily small amplitude, as we didn't take any linearize approximations) could be produced due to prominence filament's vertical motion through the uniform background magnetic field. These waves are mostly localized (as for $y > V_A t$ implies $F(\xi) = 0$ and $G(\xi) = 0$, i.e. no waves can propagate beyond $V_A t$ distance from the prominence axis).

4. Conclusions

Most of the observational data only give us a range of velocities, but not a precise value. As mentioned in the earlier section, these values and ranges vary from observation to observation and sometimes overlap. Hence, a very accurate measurement of the prominence velocity is to be conducted. This lays the foundation for validating the result for accuracy and further improvement. Moreover, as mentioned in the previous section, we match the velocities with previous observational work by physicists and the results are satisfactory. The earlier quote of a result by *B. Schmieder* et. al. puts the prominence velocity in a range up to 6 kms^{-1} [29]. An observation by *Chifor* et.al. plotted the heights of the uppermost point of the filament from 171 images from TRACE. Their findings reveal a prominence eruption with a velocity of $4.8kms^{-1}$ [3]. This observation is close to our obtained result ($5.1kms^{-1}$). Our MHD calculations obtained analytically unfold the possibility of local Alfven waves which could be produced by the fall of prominence through the background uniform magnetic field perpendicular to the prominence axis. The predictions made by our model are promising and govern the dynamics of prominence as observed. Further research and calculations may be required to precisely predict and work out the dynamics of prominence. This sets the baseline for future research.

5. Acknowledgement

We are grateful to Dr. B.C.Low (HAO, Boulder) for his insightful suggestions. The communicating author wishes to acknowledge the support received from The Inter-University Centre for Astronomy and Astrophysics (IUCAA), Pune, India during the time spent as visiting associate in the academic year 2016-17.

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Figure1

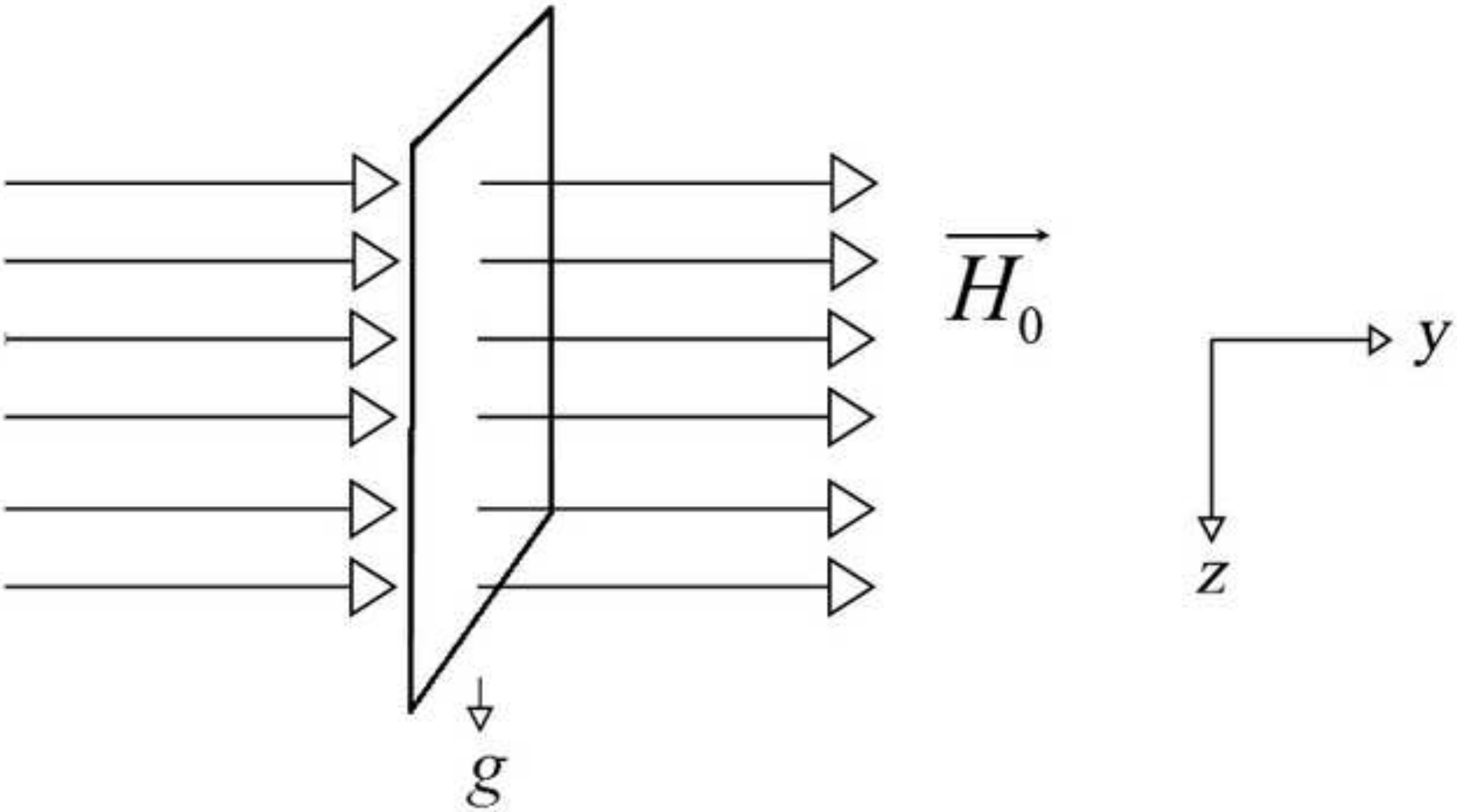
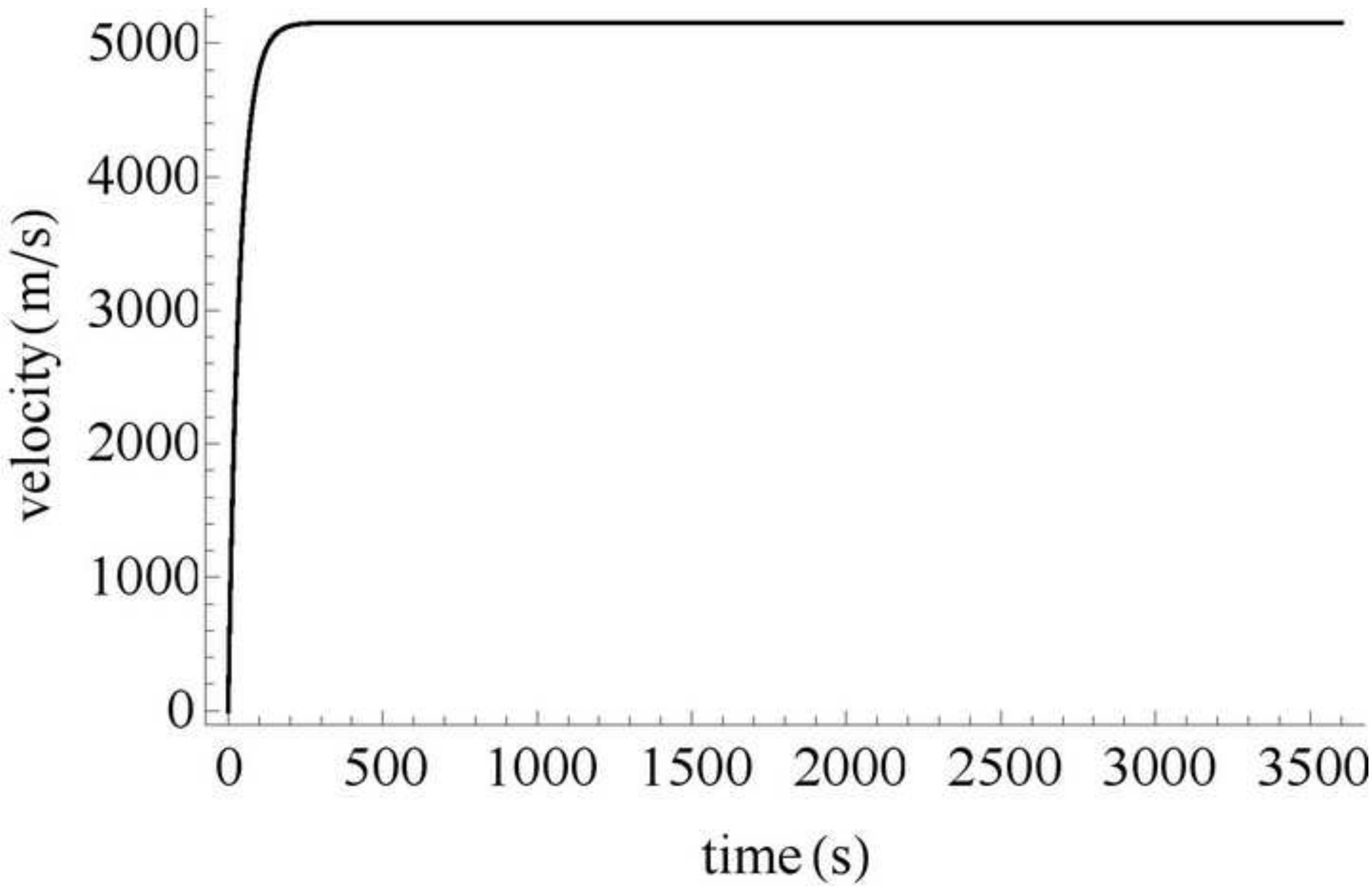


Figure2



velocity (m/s)
5000
4000
3000
2000
1000
0
0
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3500
time (s)